

# Aligned Ellipses

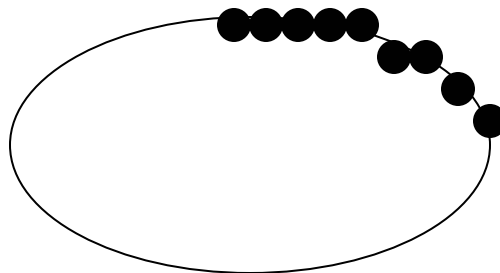
- Equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

i.e.,

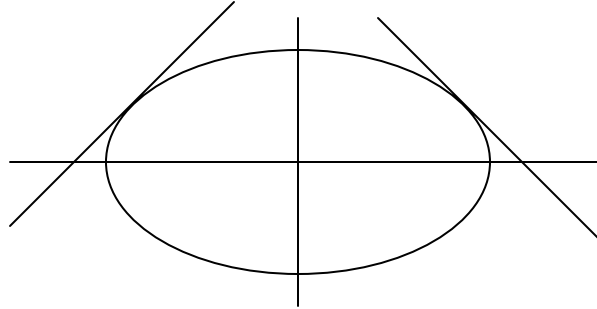
$$b^2 x^2 + a^2 y^2 = a^2 b^2$$

- Computation of  $\Delta_E$  and  $\Delta_{SE}$  is similar
- Only 4-fold symmetry
- When do we stop stepping horizontally and switch to vertical?



## Direction Changing Criterion (1/2)

- When the absolute value of the slope of the ellipse is more than 1, viz:



- How do you check this? At a point  $(x,y)$  for which  $F(x,y) = 0$ , a vector perpendicular to the level set is  $\nabla F(x,y)$  which is

$$\left[ \frac{\partial F}{\partial x}(x, y), \frac{\partial F}{\partial y}(x, y) \right]$$

- This vector points more right than up when

$$\frac{\partial F}{\partial x}(x, y) - \frac{\partial F}{\partial y}(x, y) > 0$$

## Direction Changing Criterion (2/2)

- In our case,

$$\frac{\partial F}{\partial x}(x, y) = 2a^2 x$$

and

$$\frac{\partial F}{\partial y}(x, y) = 2b^2 y$$

so we check for

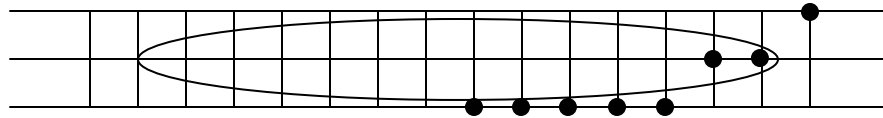
$$2a^2 x - 2b^2 y > 0$$

i.e.

$$a^2 x - b^2 y > 0$$

- This, too, can be computed incrementally

# Problems with Aligned Ellipses



- Now in ENE octant, not ESE octant
- This problem is due to *aliasing* – much more on this later